Mathematical Misnomers

Hey, who really discovered that theorem!

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NEWTON 1642-1727

Who was buried in Grant's tomb?

- Ulysss S. Grant, of course!
- Sandwiches were invented by the Earl of Sandwich.
- And Cramer's rule was really discovered by Cramer.

But not all things are as they seem....

Inverse-square law

- Kepler (b1471) discovered the planetary ellipses.
- But who first thought of the inverse-square law?
- Robert Hooke (b1635) in letters to Newton, circa Dec. 1679.
- Newton (N 37) wrote that a dropped ball falling to the center of the Earth would wind in a spiral.
- Hooke said, No, and guessed an ellipse, tugged by an inverse-square law—but offered no theory!

The hard part was....

- To derive planetary motion—**from first principles**!
- *That's* what Newton did...
  by first formulating the laws of mechanics and inventing his version of calculus called *fluxions*.
- Newton **proved** that elliptical orbits imply an inverse-square law, and vice-versa.
- But he didn't formulate and solve the differential equations of motion—Johann Bernoulli (b1667) did that!

"Jakob (b1664) Bernoulli's" summation formula

- $$1^k + 2^k + \cdots + n^k =$$

$$\frac{n^{k+1}}{k+1} + \frac{n^k}{2} + \frac{1}{2}\binom{k}{1} B_2 n^{k-1} + \frac{1}{4}\binom{k}{3} B_4 n^{k-3} + \cdots + B_k n, \ n > 1$$

- Essentially discovered decades earlier by Johann Faulhaber (b1580), the "Calculating Wizard of Ulm."
- The *B*s were named "Bernoulli numbers" by Euler (b1707)— some say by de Moivre (b1667):

$$B_1 = -\frac{1}{2}, \ B_2 = \frac{1}{6}, \ B_4 = -\frac{1}{30}, \ B_6 = \frac{1}{42}, \ldots$$

$$B_3 = B_5 = B_7 = \cdots = 0$$

Euler's use of Bernoulli numbers

- Euler (b1707) became famous by solving the "Basel problem" (posed by Mengoli in 1644)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots =? = \frac{\pi^2}{6}$$

- Later he found

$$\frac{1}{1^{2k}} + \frac{1}{2^{2k}} + \frac{1}{3^{2k}} + \cdots = (-1)^{k+1} \frac{2^{2k-1}\pi^{2k}}{(2k)!} B_{2k}$$

- And he derived a *generating function* for the Bernoulli numbers

$$\frac{x}{e^x - 1} = B_0 + \frac{B_1}{1!}x + \frac{B_2}{2!}x^2 + \frac{B_4}{4!}x^4 + \cdots$$

An unsolved problem

- Euler tried but failed to find simple formulas for series of **odd** powers, like

$$\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots$$

  And nobody else has ever succeeded, either.

- But Euler did find formulas for alternating series of odd powers, like

$$\frac{1}{1^5} - \frac{1}{3^5} + \frac{1}{5^5} - \frac{1}{7^5} \pm \cdots$$

Jacob Bernoulli's probability theory (Ars Conjectandi)

- The probability of k heads in n tosses is

$$\binom{n}{k} p^k (1-p)^{n-k} \quad (\texttt{Bernoulli's distribution})$$

- What does Bernoulli's distribution look like for large n? (For $p = 1/2$ you can get an idea by looking at Pascal's (b1623) triangle.)
- To figure it out you need to estimate

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

for large n and k.

"Gauss's" (b1777) normal distribution—the bell curve

- In Bernoulli's distribution, substitute "Stirling's" (b1692) formula

$$m! \approx \sqrt{2\pi}\, m^{m+\frac{1}{2}} e^{-m}$$

- Then re-scale Bernoulli's distribution (horizontally and vertically) to obtain the limiting distribution:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \; (\texttt{Normal distribution})$$

- It was **de Moivre** (b1667) who discovered "Stirling's" formula on his way to discovering "Gauss's" normal distribution.

How did de Moivre do it?

- To derive “Stirling’s” formula, de Moivre used “Maclaurin’s” (b1698) summation formula: if $a_j = f(j)$ for differentiable $f(x)$, then

$$\sum_{j=0}^{n} a_j = \int_0^n f(t)dt + \frac{1}{2}f(n) +$$

$$\sum_{j=1}^{m} \frac{B_{2j}}{(2j)!}\left[f^{(2j-1)}(n) - f^{(2j-1)}(0)\right] + R_m$$

- Yes, the *B*s are the Bernoulli numbers again!
- And the formula was discovered first by **Euler**, so to be fair it is now called *Euler’s summation formula* or the *Euler-Maclaurin summation formula*.

So, who invented calculus?

- Newton and Leibniz (b1646), right? Well, yes, but ....
- **Newton**:

  De methodis serierum et fluxionum, 1670-71 (N 28-29)

  Philosophiae Naturalis Principia Mathematica, 1687 (N 45)

  Newton used geometric methods and fluxions, which we do not use.
- **Leibniz**:

  Nova methodus...., 1684 (N 42)

Differential and integral calculus—as we know it

- L'Hospital's book *Analysis of The Infinitely Small, 1696* (based on Johann Bernoulli's lectures) popularized Leibniz's approach to differential and integral calculus.
- In that tradition **Euler** (b1707) developed calculus in terms of *functions*, *infinite series*, *differential equations*, *calculus of variations*, and in laying foundations for *analytic mechanics of solids, fluids and elastic media*....
- Modern rigor came later in the work of Cauchy (b1789), Weierstrass (b1815), Riemann (b1826), Dedekind (b1831) and Cantor (b1845).

And, Oh, yeh,

- “L’Hospital’s” rule was discovered by Johann Bernoulli!!

"Stokes' " Theorem

- $\int_{\partial S} \mathbf{F} \cdot \mathbf{t}\, ds = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$
- You guessed it: it's not really Stokes's theorem!
- It was discovered by his friend, the Scotch-Irish physicist William Thompson— knighted Lord Kelvin (b1824).

  Stokes used to pose it as a problem on a famous Cambridge Math contest—the *Tripos*.

Closing thought

If you happen to be a discoverer who's name is forgotten, you will have had all the fun making the discovery anyway.

Isn't that the best part?

Recommended reading

Easy reads:

- *Euler, Master of us All* by Dunham
- *A Very Short Introduction to Newton* by Iliffe
- *The Calculus Gallery* by Dunham

More on methods of Newton, Leibniz and Euler:

- *Reading the Principia* by Guicciardini
- *Theory and Application of Infinite Series* by Knopp
- *Basic Calculus from Archimedes to Newton* by Hahn